OPTIMAL DECOMPOSITION FOR WAVELET IMAGE COMPRESSION

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Abstract – *The paper discusses important features of wavelet transform in compression of still images including the extent to which the quality of image is degraded by process of wavelet compression and decompression. A set of wavelet functions (wavelets) for implementation in still image compression system is examined. The effects of different wavelet functions, image contents and compression ratios are assessed. The benefit of this transform relating to today's methods is stressed. Our results provide a good reference for application developers to choose a good wavelet compression system for their application.*

Key words – *Image Coding, Transform Coding, Wavelet Transforms*

I. INTRODUCTION

In recent years many studies have been made on wavelets. Excellent overview of what wavelets have brought to the fields as diverse as biomedical applications, wireless communications, computer graphics or turbulence, is given in (Proceedings, 1996). Image compression is one of the most visible applications of wavelets.

A typical still image contains a large amount of spatial redundancy in plain areas where adjacent picture elements (pixels, pels) have almost the same values. In addition, still image can contain subjective redundancy, which is determined by properties of human visual system (HVS) (Jayant, 1993). HVS presents some tolerance to distortion depending upon the image content and viewing conditions. The redundancy (both statistical and subjective) can be removed to achieve compression of the image data. The basic measure for the performance of a compression algorithm is compression ratio defined as ratio between original data size and compressed data size. In lossy compression scheme, image compression algorithm should achieve trade off between compression ratio and image quality (Zovko-Cihlar, 1995). A standard objective measure of compressed image quality is peak signal to noise ratio (PSNR). For the common case of 8 bits per picture element of input image, PSNR is defined as the ratio of maximum input symbol energy to mean square error (MSE) which evaluate difference between input image and reconstructed image, (1).

$$PSNR(dB) = 10\log_{10}\left(\frac{255^2}{MSE}\right) \qquad (1)$$

Transform coding is a widely used method of compressing image information. In a transform based compression system two-dimensional images are transformed from the spatial domain to the frequency domain. An effective transform will concentrate useful information into a few of the low frequency transform coefficients. HVS is more sensitive to energy with low spatial frequency than with high spatial frequency. Therefore compression can be achieved by quantizing the coefficients so that important coefficients (low frequency coefficients) are transmitted and remaining coefficients are discarded. Very effective and popular ways to achieve compression of image data are based on Discrete Cosine Transform (DCT) and Discrete Wavelet Transform (DWT).

Current standards for compression of still (e.g. JPEG (ISO, 1991)) and moving images (e.g. MPEG-1 (ISO, 1993), MPEG-2 (ISO, 1994)) uses DCT, which represents an image as a superposition of cosine functions with different discrete frequencies. DCT provides excellent energy compaction and a number of fast algorithms exist for calculating the DCT. Most existing compression systems use square DCT blocks of regular size (ISO, 1991), (ISO, 1993), (ISO, 1994). The use of uniformly sized blocks simplified the compression system but does not take into account the irregular shapes within real images. The block-based segmentation of source image is fundamental limitation of the DCT-based compression system (Bauer, 1996). The degradation is known as "blocking effect" and depends on block size.

In recent time, much of the research activities in image coding have been focused on the Discrete Wavelet Transform (DWT) which has become a standard tool in image compression applications because of their data reduction capability (Lewis, 1992), (Antonini, 1992). In wavelet compression system the entire image is transformed and compressed as a single data object rather than block by block as in DCT based compression system. It allows a uniform distribution of compression error across the entire image. DWT offers adaptive spatial-frequency resolution (better spatial resolution at high frequencies and better frequency resolution at low frequencies) that is well suited to the properties of HVS. It can provide better image quality than DCT especially on higher compression ratio (Grgic, 1999). But the implementation of the DCT is less expensive than that of the DWT. For example, the most efficient algorithm for 2-D 8x8 DCT requires only 54 multiplication (Feig, 1990) while the complexity of calculating DWT depends on the length of wavelet filters, which is at least one multiplication per coefficient.

A wavelet image compression system can be created by selecting a type of wavelet function, quantizer and statistical coder. In this paper we do not intend to give technical description of wavelet image compression system. We used a few general types of wavelets and compared effects of wavelet analysis and representation, compression ratio and image content resolution to image quality.

II. WAVELET TRANSFORM

Wavelet transform (WT) represents an image as a sum of wavelet functions (wavelets) with different locations and scales (Daubechies, 1992). Any decomposition of an image into wavelets involves a pair of waveforms: one to represent the high frequencies corresponding to the detailed parts of an image (wavelet function-Ψ) and one for the image's low frequencies or smooth parts (scaling function-Φ). The two waveforms are translated and scaled on the time axis to produce a set of wavelet functions at different locations and on different scales. The result of WT is a set of wavelet coefficients, which measure the contribution of the wavelets at these locations and scales.

Multiresolution Analysis

WT performs multiresolution image analysis (Mallat, 1989). The result of multiresolution analysis is simultaneous image representation on different resolution (and quality) levels (Mallat, 1989). The resolution is determined by threshold below which all fluctuations or details are ignored. Difference between two neighbouring resolutions represents details. Therefore an image can be represented by low-resolution image (approximation or average part) and the details on each higher resolution level.

DWT for an image as a two-dimensional (2-D) signal can be derived from one-dimensional (1-D) DWT. Easiest way for obtaining scaling and wavelet function for two-dimensions is by multiplying two 1-D functions. Scaling function for 2-D DWT can be obtained by multiplying two 1-D scaling functions: $\Phi(x,y)=\Phi(x)\Phi(y)$. Wavelet functions for 2-D DWT can be obtained by multiplying two wavelet functions or wavelet and scaling function for 1-D analysis. For 2-D case there exists three wavelet functions that scan details in horizontal $\Psi^{(I)}(x,y)=\Psi(x)\Phi(y)$, vertical $\Psi^{(II)}(x,y)=\Phi(x)\Psi(y)$ and diagonal direction: $\Psi^{(III)}(x,y)=\Psi(x)\Psi(y)$. As result, there are three types of detail images for each resolution: horizontal, vertical and diagonal. The decomposition can be repeated on the average part of image. Here we adopt the term "number of decompositions - D" to describe this process. Thus, a typical 2D-DWT, used in image compression, will generate hierarchical pyramidal structure. Fig. 1. shows three levels of wavelet decomposition of image "Lena" (D=3).

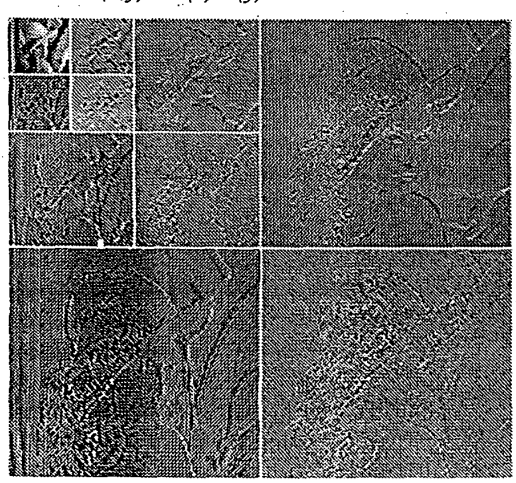


Fig. 1. Pyramidal structure of a wavelet decomposed image "Lena" (D=3)

Wavelet multiresolution and direction selective decomposition of images is matched to

HVS (Mallat, 1996). In the spatial domain the image can be considered as composition of information on a number of different scales. A wavelet transform measures gray level image variations at different scales. In the frequency domain the contrast sensitivity function of the HVS depends on frequency and orientation of the details.

III. DWT IN IMAGE COMPRESSION

Image Contents

The fundamental difficulty in testing image compression system is how to decide which test images to use for the evaluations. The image contents being viewed influences the perception of quality irrespective of technical parameters of the system. Normally, a series of pictures, which are average in terms of how difficult they are for system being evaluated, has been selected. To obtain a balance of critical and moderately critical material we used four types of test images with different frequency content: Peppers, Lena, Baboon and Zebra. Spectral activity of test images is evaluated using DCT applied to the whole image. DCT coefficients as result of DCT show frequency contents of the image. Fig. 2. shows the distributions of image values before and after DCT. The distribution of DCT coefficients depends on image contents (white dots represent DCT coefficients, arrows indicate the increase of horizontal and vertical frequency). Moving across the top raw, horizontal spatial frequency increases. Moving down vertical spatial frequency increases. Images with high spectral activity are more difficult for compression system to handle. These images usually contain large number of small details and low spatial redundancy.

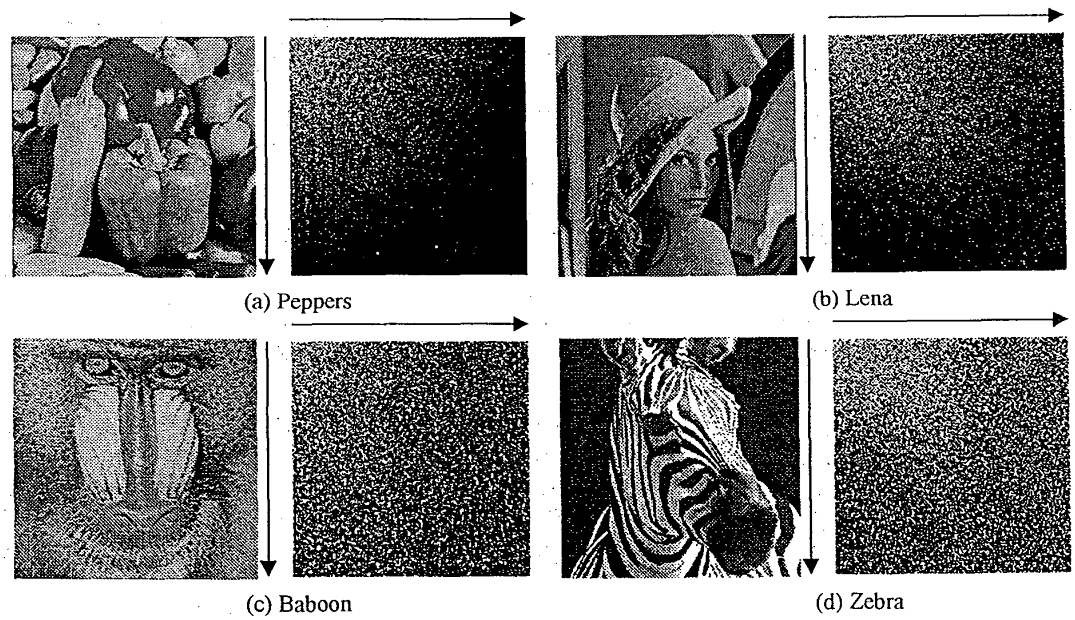


Fig. 2. Frequency contents of test images

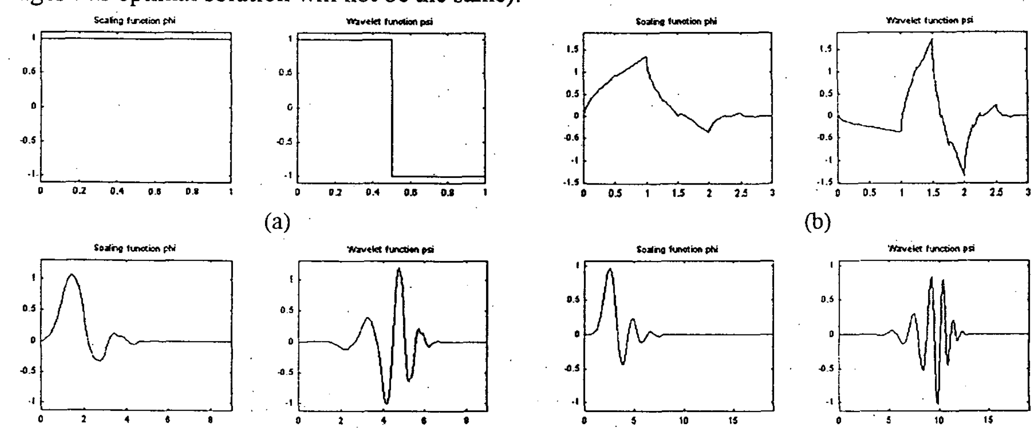
Choice of Wavelet Function

Choice of wavelet function is crucial for coding performance in image compression. This choice should be adjusted to image content. Important properties of wavelet functions in image compression applications are compact support (lead to efficient implementation), symmetry (useful in avoiding dephasing in image processing), orthogonality (allow fast algorithm), regularity and degree of smoothness (related to filter order or length of wavelet filter).

In our experiment four types of wavelet families are examined: Haar Wavelet (HW), Wavelet (DW), Coiflet Wavelet (CW), and Biorthogonal Wavelet (BW). Daubechies and Coiflet wavelets are families of orthogonal wavelets that are compactly supported. Compactly supported wavelets correspond to finite impulse response (FIR) filters and thus lead to efficient implementations (Zettler, 1990). A major disadvantage of these wavelets is their asymmetry, which can cause artifacts at borders of the wavelet subbands. DW is asymmetrical while CW is almost symmetrical. Symmetry in wavelets can be obtained only if we are willing to give up either compact support or orthogonality of wavelet (except for Haar wavelet, which is orthogonal, compactly supported and symmetric). If we want both symmetry and compact support in wavelets, we should relax the orthogonality condition and allow nonorthogonal wavelet functions. The example is the family of biorthogonal wavelets that contains compactly supported and symmetric wavelets (Cohen, 1992).

Filter Order (Length of Wavelet Filter)

Each wavelet family can be parameterised by an integer N that determines filter order. In our examples different filter orders are used inside each wavelet family. The length of wavelet filter is determined by filter order but relationship between filter order and length of wavelet filter is different for different wavelet families. For example, the filter length is 2N for DW family and 6N for CW family. Haar wavelet is the special case of DW with filter order 1 (DW-1). Biorthogonal wavelets can use filters with similar or dissimilar orders for decomposition (Nd) and reconstruction (Nr). Therefore BW is parameterised by two numbers and filter length is {max(2Nd, 2Nr)+2}. Higher filter orders give wider functions in time domain with higher degree of smoothness, Fig. 3. Wavelet based image compression prefers smooth functions of relatively short support. So, in image compression application we have to find balance between length of wavelet filter and degree of smoothness. Inside each wavelet family we can find wavelet function that represents optimal solution related to length of wavelet filter and degree of smoothness but this solution depends on image contents (for different images this optimal solution will not be the same).



(a) (b)

(c) (d)

Fig. 3. DW family-scaling and wavelet functions for different filter orders: (a)N=1; (b)N=2; (c)N=5, (d)N=10

Number of Decompositions

The quality of compressed image depends on the number of decompositions D. After decomposing image and representing it with wavelet coefficients, compression can be performed by ignoring all coefficients below some threshold. Number of decompositions determines the resolution of the lowest level in wavelet domain. If we use larger D, we will be more successful in resolving important DWT coefficients from less important coefficients. Human visual system is less sensitive to removal of smaller detail. On the other hand, larger D causes the loss of the coding algorithm efficiency. Therefore, adaptive decomposition is required to achieve balance between image quality and complexity of computations.

Optimal D depends on filter order N. Higher N does not imply better image quality because of the length of wavelet filter, which becomes limiting factor for decomposition. Decisions about the filter order and number of decompositions are matter of compromise.

IV. DWT COMPRESSION RESULTS

The choice of optimal wavelet function in image compression system for different image types can be provided in few steps. For each filter order in each wavelet family optimal D can be found. Optimal D gives the highest PSNR values in the wide range of compression ratios for given N. Table 1 shows some of the results for DW and image "Lena". For lower filter orders better results are reached with more decompositions. Shaded areas in Table 1 show optimal D for given N while bold types mark optimal combination of N and D for image "Lena" (D=5, N=5). Similar results were achieved for other wavelet families and other test images. For each wavelet family different filter orders are tested using different test images. For each test image and each wavelet family optimal combination of filter order and number of decompositions was found (shaded areas in Table 2).

Table 1 – Optimal number of decompositions for different filter orders in DW family

Wavelet		Compression Ratio				D
Family	N	5:1	30:1	50:1	100:1	
DW	1	35,53	14,76	11,53	9,42	2
		36,29	25,68	23,79	21,59	4
		36,39	25,74	23,94	21,87	6
		36,28	25,75	23,96	21,94	8
	2	36,79	15,09	11,73	9,51	2
		37,25	25,86	23,86	21,36	4
		37,18	25,79	23,89	21,52	6
		37,18	25,72	23,73	21,33	8
	5	37,04	15,23	11,73	9,51	2
		37,16	26,21	24,46	22,09	4
		37,05	**26,24**	**24,53**	**22,18**	**5**
		36,98	26,13	24,37	22,13	6
		36,88	25,91	24,17	21,82	8
	10	35,65	15,15	11,78	9,52	2
		35,72	25,49	23,85	21,81	4
		35,47	24,13	23,45	21,41	6
		35,08	24,47	22,67	20,38	8

Table 2 – PSNR results in (dB) for different wavelet families and different compression ratios

Wavelet		Peppers			Lena			Zebra			Baboon			D
Family	N	5:1	50:1	100:1	5:1	50:1	100:1	5:1	50:1	100:1	5:1	50:1	100:1	
DW	1	37,98	23,91	21,72	36,28	23,96	21,94	29,16	17,26	15,90	28,01	21,84	21,07	8
	3	39,31	24,30	21,70	37,33	24,26	22,14	28,06	17,75	16,25	17,98	21,68	20,87	6
	5	38,75	24,24	21,69	37,05	24,53	22,18	27,72	17,81	16,11	28,21	21,76	20,77	5
	10	37,09	23,05	16,60	36,00	23,20	16,46	27,02	16,84	14,44	27,59	21,15	12,57	3
CW	2	39,67	24,62	22,18	37,80	24,55	22,33	28,30	17,92	16,44	28,22	21,82	20,95	5
	3	39,65	24,82	22,29	37,73	23,54	22,49	28,10	17,95	16,27	28,24	21,85	20,87	4
	4	38,96	24,56	21,81	37,52	24,50	22,15	27,96	17,93	1617	28,23	21,75	20,77	4
	5	38,92	24,57	21,94	37,46	24,64	22,43	27,90	17,97	16,30	28,24	21,76	20,86	4
BW	2.2	40,20	24,67	22,09	37,78	24,69	22,51	28,22	17,56	16,01	27,71	21,66	20,88	6
	3.1	37,93	21,62	18,39	35,36	21,29	18,52	24,22	14,65	12,56	24,66	16,67	17,63	6
	3.3	39,15	23,30	20,78	36,54	23,38	21,11	26,24	16,72	15,20	26,39	20,70	20,06	5
	6.8	39,62	23,09	16,28	37,98	23,25	16,39	28,10	16,75	14,28	28,18	21,07	12,65	3

The filter orders which give the best PSNR results inside each wavelet family are different for different test images except for BW family where filters with order 2 in decomposition and order 2 in reconstruction (BW-2.2) gives the best results for all image types. The comparison of PSNR values of optimal filters (shaded areas in Table 2) from each wavelet family for different test images shows that image "Peppers" (low spectral activity) has highest PSNR values and image "Zebra" (high spectral activity) has smallest PSNR values. PSNR values depend on image type and cannot be used if we want to compare images with different contents. If we want to compare visual quality of different test images PSNR is not adequate measure and other measures such as Mean Opinion Score (MOS), (Grgic, 1999) and Picture Quality Scale (PQS), (Miyahara, 1996) should be used.

V. CONCLUSIONS

We presented results from a comparative study of different wavelet based image compression systems using objective PSNR quality measures. Wavelet-based image coding prefers smooth functions of relatively short support with some degree of regularity. A suitable number of decompositions should be determined by means of image quality and less computational operation on the reconstruction process. Optimal number of decompositions depends on filter order and image contents. Decisions about the filter order and number of decompositions are matter of compromise. Optimal wavelet function for some image compression application should be adjusted to the typical image contents. The entire compression system should be designed with considerations of the characteristics of HVS.

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